

Chinese Young Children's Strategies on Basic Addition Facts

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Kindergartens in China offer structured full-day programs for children aged 3-6. Although formal schooling does not commence until age 7, the mathematics program in kindergartens is specifically focused on developing young children's facility with simple addition and subtraction. This study explored young Chinese children's strategies for solving basic addition facts as well as their intuitive understanding of addition via interview methods. Results indicate a strong impact that teacher-directed teaching methods have on young children's cognitions in relation to addition.

Introduction

In ancient China, mathematics was called "Suan Xue", which means "the knowledge of computations" (Zhang, 1998). Thus the Chinese mathematics curriculum predominantly focuses on mathematics as calculation. As a result, Chinese teachers are very interested in teaching faster and easier ways to do given computations (Ma, 1999). Therefore, performing calculations mentally is strongly emphasized in Chinese mathematics classes due to the speed at which calculations can be performed over other methods.

The abacus is a tool to help students with mental calculation (Zhang, 1998). It becomes a fashion that many kindergarten classrooms in China begin to teach children Mathematics with the help of an abacus. An abacus consists of columns of beads. The beads are separated as upper part and lower part by a crossbar. Each column has one bead above the crossbar and four beads below it. Each lower bead represents one unit and each bead above represents five units.

The abacus facilitates speed of calculation. According to Zhang (1998), Chinese students spent 5 seconds to complete an addition calculation for two whole numbers with two digits, and 14.5 seconds for multiplication (Zhang, 1998).

Literature Review

Developing Addition

Research into the development of addition. Addition is an important concept in mathematics education. It is the first algorithm learned by children and it is also fundamental to children's later learning in mathematics (Braten, 1998; Siegler, 1987). An understanding of the principles of additive composition by which parts are combined to form a whole is the conceptual core of the mathematics curriculum in the early elementary school period (Resnick, 1989).

The development of addition has been shown in sequential stages, and different levels of strategies have been identified either by chronometric research (Groen & Parkman, 1972; Suppes & Groen, 1967) or interview research (Baroody, 1987; Carpenter & Moser, 1984; Siegler & Jenkins, 1989), as children have solved single digit addition facts with two addends.

When computing basic addition facts, counting strategies and non counting strategies are found to be employed by young children. Counting strategies are identified by many researchers (e.g., Groen & Parkman, 1972; Suppes & Groen, 1967) as counting all by sum, counting all from the first addend, counting all from the second addend, counting on from the smaller addend and counting on from the larger addend (termed as Min strategy by Parkman & Groen, 1971). Carpenter and Moser (1982) further classified those counting strategies into counting with models such as fingers or other physical objects, which were also identified as concrete counting strategies in Baroody's (1987) study; and counting without models.

Non counting strategies include guessing (Siegler, 1987), known fact and derived facts (Carpenter & Moser, 1982). Known facts and derived facts strategy were termed as retrieval and decomposition strategies in Siegler's (1987) study respectively. Known fact referred to the number facts that children already knew and could recall instantly, while derived facts were the facts children derived by linking to a known number fact (Carpenter & Moser, 1982).

Baroody's (1987) study suggests that there is a developmental sequence based on efficiency, and possibly greater understanding, and familiarity with numbers and number properties. Counting on from the first is the shortcut of counting all starting with the first addend. Counting all starting with the larger addend and counting on from the larger addend minimises the keeping-track process. Counting on starting from the first is the shortcut of counting all starting with the first addend. Counting on starting from the larger addend short cuts the procedure of counting all starting with the larger addend.

Siegler (1987) also found that there were clear developmental trends in children's strategies. He contended that, as children became more proficient with addition, the use of retrieval and decomposition strategies increased and the use of counting all and guessing decreased.

In studies on young children's early addition strategies, three models appear to be the most frequently identified: counting all by sum (Sum model), counting on by holding the first addend constant, and counting on from the larger addend (Min model) (Carpenter & Moser, 1984; Houlihan & Ginsburg, 1981). The Min model is considered to be the most economical and advanced counting strategy in addition (Braten, 1998).

Strategy choice and variability. Although many researchers have suggested that addition models/strategies used by children to solve addition problems are hierarchical, children do not use the most efficient strategy at all times (Carpenter & Moser, 1984; Christensen & Cooper, 1991; Houlihan & Ginsburg, 1981). For example, Carpenter and Mose found that children did not always use the most efficient strategy, though they had known several strategies. Even when children mastered several strategies, they did not use the most efficient one consistently but would use them interchangeably, and in some circumstances, they would often revert to a less efficient strategy.

Many researchers agree that addition strategies used by children will vary according to their ages and background knowledge (e.g., Siegler, 1987; 2003; Siegler & Campbell, 1989; Siegler & Jenkins, 1989). In other words the strategies that a child may use will depend not only on the child's age, but also on how the child builds new strategies on their existing knowledge. Strategy use has also been shown to depend on the type of addition problem. Siegler (1987) summarized the conditional probabilities of strategy use, stating that children used retrieval to solve the problems with small sums and "ties" or "doubles" (e.g., $5+5$) because these problems could be easily executed to obtain correct answers. The Min strategy (Counting on from the large) is often used when children solve problems containing small addends or where there are large differences between addends, or both. On solving problems with small differences between addends, children use the Counting-All strategy. Guessing is used most often on problems with large sums. More recently, Carr and Jessup (1994) have suggested that children's metacognitive knowledge is significantly correlated with their use of correct strategies. And further, current research indicates that strategy use/choices is facilitated by number sense (Griffin, 2003).

Developing Basic Facts

According to Isaacs and Carroll (1999) one of the objectives of elementary mathematics instruction is to teach basic number facts. Basic number facts underpin mental computation facility (McIntosh & Dole, 2000).

Isaacs and Carroll have proposed a "strategies-based approach" for teaching basic facts that differs from a traditional rote approach that may discourage the development of children's mathematical thinking. Their approach is based on building children's informal mathematical knowledge, and helping them develop their immature addition strategies into more mature strategies.

Many researchers (e.g., Isaacs & Carroll, 1999; Johnson & Siegler, 1998; Postlewait, Adams, & Shih, 2003) have been trying to find a meaningful way to teach children addition and subtraction in the early childhood classroom. All these approaches highlight that instruction should be based on the informal mathematical knowledge children bring to school, and build on children's number sense. However, Chinese kindergarten children appear to learn mathematics in a very formal environment and they do well in achievement tests. As there is a large body of literature on elementary mathematics that focuses on children who are aged 6 and above, and many cross-nation studies conducted detailing the reasons why Chinese students outperform their peers, the aims of the study are to investigate young (aged 3-6 years) Chinese children's basic addition fact strategies during their three years of formal kindergarten education, and to examine the impact of formal teaching upon kindergarten children's choice of addition strategies.

Methodology

Participants

Seventy-two children, aged from 3 to 6 years, enrolled in a public Kindergarten in Baoding, a middle sized city in China, participated in this study. The children were grouped by year level: Group K1 consisted of children in their first year of kindergarten; Group K2 consisted of children in their second year of kindergarten, and Group K3 consisted of children in their third year of kindergarten. The specific number, gender and mean age of the children within each of these three groups are displayed in Table 1.

Table 1

Number, Gender and Mean Age of the Children within Each of Three Groups in the Study

Kindergarten Year	Number of Boys	Number of Girls	Mean Age
K1	11	11	3 years, 5 months
K2	13	9	4 years
K3	15	13	5 years, 5 months

Instruments

The basic addition facts test was used to assess children's basic addition fact knowledge and the strategies they used. The test contained 40 addition problems, grouped into 6 categories as follows: Adding on 1, 2, and 3, Adding 0, Bridging 10, Near Doubles, Doubles, and Tens facts. Children were required to give the answer orally and to explain the strategy they used to arrive at the answer to each item.

Procedure

Step 1 – Interviews with children. Each child was tested on the Basic Addition Facts Test. All children were taken individually to a separate room that was attached to their classroom to do the test.

The addition problems were presented one at a time on cards, and the children were encouraged to use whatever strategy made it easiest for them to arrive at the answer. They were encouraged to speak aloud their thinking and were told to keep talking as long as they could. The strategies were classified as counting all by sum, counting on from the first addend, counting on from the larger addend, min, abacus, recall, visualising abacus, and guessing.

Step 2 – Classroom observation. Observations took place over a period of two weeks for each year level of kindergarten as the teacher implemented the planned curriculum for addition. The researcher and one trained observer observed each classroom. During these observations, teacher and children's interactions were video taped, and notes were taken, which described what children were doing and general impressions of the class. The researcher watched and reviewed the tapes of the teachers, adding information from field notes until fairly complete records were compiled.

Findings

Mathematics Class Observation

The abacus was first introduced when children were in K1; K2 children started to have more experience with the abacus and children were overtly given instructions and lots of drills on using the abacus. It seemed that most of children had become very proficient with the abacus in K3.

The Mathematics learning outcomes for K1 Children is developing one to one correspondence and number combinations up to 5. Children were also introduced to the abacus in K1. They were expected to learn how to present numbers from 1 to 5 on the abacus by arranging the beads in the appropriate form.

The Mathematics outcome for K2 children is mastering combinations up to 10. Additional lessons on the abacus were conducted. Children learned the rules on how to use the abacus for doing simple addition (See Figure 1 & 2).

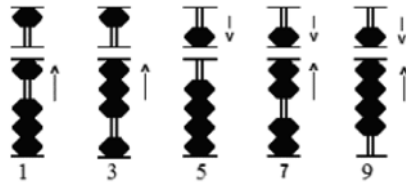


Figure 1. Some examples of numbers shown on an abacus. (Adapted from Heffelfinger & Flom, 2004)

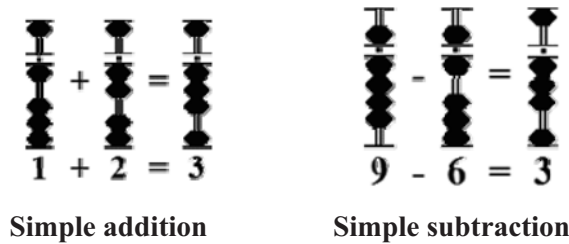


Figure 2. Simple addition and subtraction by using the abacus. (Adapted from Heffelfinger & Flom, 2004)

When there are not enough beads on the column to complete addition or subtraction, the technique employed is the use of combinations (complementary numbers) of 5 and 10 (Heffelfinger & Flom, 2004). Therefore, children have to be proficient with combinations of 5 and 10. In doing addition, one always subtracts the complementary numbers. Two groups of complementary numbers (4 & 1 and 3 & 2) are used for number 5. Five groups of complementary numbers are for number 10 (9 & 1, 8 & 2, 7 & 3, 6 & 4, and 5 & 5). For example, in the case of $4+8$, one has to push up 4 beads on the column B first. As there are not enough beads on Column B to represent 8, the complementary number is used. Two is 8's complementary number in making 10. Then add 8 by subtracting the complementary number 2 and carry one to Column A (See Figure 3).

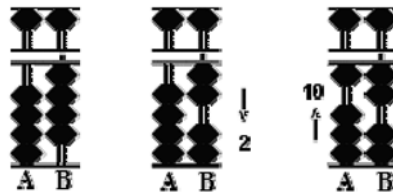


Figure 3. The action on an abacus for $4+8=12$. (Adapted from Heffelfinger & Flom, 2004)

At K3, children were very familiar with abacus. They even learned how to compute addition and subtraction with three digits. Many practices on abacus were given. In the observed lessons, the children were usually given a group of similar problems to solve until they mastered them. In the case of learning $4+8$, children would give similar problems such as $4+7$, $3+8$, $2+8$, $9+7$ and so on, to practice until they mastered them.

The idea of using the abacus was to eventually help children to transform mechanically adding and subtracting beads into visualised abacus in their minds. At K3 stage, children were continually encouraged to visualise the abacus in their mind when doing computation. Children were found to physically use the abacus only when the numbers were bigger and were beyond their ability to reach the answer automatically. Counting with fingers was actively discouraged by the teacher.

Strategy Use to Solve Basic Addition Facts by Each Year Level

At K1 level, where children were 3 to 4 years old and in their first year of kindergarten, only a few children attempted to compute all the addition tasks in the test. In total the overall success on the test was 20%. The majority of K1 children used a guessing strategy, or made no attempt. This finding echoes level 1 strategy in Griffin's (2003) study with kindergarten children. Griffin found most of the children in the sample at the

age of 3 to 4, responded with blank expression or said they did not know the answer instead of making any attempts to solve the problem $4+3$. However, results of this study also show that K1 children used a range of strategies including sum, min, guessing strategies to solve basic addition facts. There was evidence of recall, and visualisation of the abacus. In this study, many K1 children easily recalled $1+1$ (86%), and some recalled $2+2$ (32%) but not other facts. These K1 children relied on Sum strategies even sometimes they could not arrive at the right answers. Only one child was found to visualise the abacus.

At 4 to 5 years of age, children in K2 were in their second year of kindergarten. Like K1 children, results of the basic facts test show that K2 children experience difficulties when they compute basic addition facts where the addends are bigger than 5. Yet overall, they could fluently compute (recall) the facts with addends smaller than 5. The mean of correct answers for the basic facts test was 79.6%.

Nine different addition strategies were used by K2 children in this study: recall, derived fact, represented or visualised addends and recalled fact, counting on, min, sum by materials, sum by fingers, guessing, and visualising the abacus. However, the strategy most used by the children was the sum strategy either with the help of fingers or materials. This supports prior research findings that children don't use the most efficient strategies all the time (Carpenter & Moser, 1984; Christensen & Cooper, 1991; Houlihan & Ginsburg, 1981). Although K2 children used a range of strategies, most of them had many facts automated (i.e., they used a recall strategy), and there was 79.6% mastery from these very young children. The other developing strategy identified was the visualising abacus strategy that was used by many more K2 children than K1 children.

At 5 to 6 years of age, children in K3 were found to use six strategies for addition facts: recall, Derived facts, Represented or visualised addends and recalled fact, visualised abacus, min (count in mind), and abacus. Children in K3 had a high level of mastery of the basic facts, with a group mean of 99% on the basic facts test. Recall strategy (48%) was found to be the dominant strategy used by K3 children and the second dominant strategy was Visualised Abacus (31%). Children frequently used visualised abacus or abacus strategy when they computed facts especially, with larger addends (e.g., Bridging 10, Near Doubles or Doubles).

In the basic facts test, there are six categories of addition facts. They are (1) Adding on 1, 2, 3 facts, (2) Adding 0 facts, (3) Bridging 10 facts, (4) Near double facts, (5) Doubles facts, and (6) Tens facts. In this study, K1 children spent the least time on Doubles facts and the most time on Bridging 10 Facts. For the Adding on 1, 2, 3 facts, six main strategies were found to be used by K1 children, and 23% of the children could successfully reach the answers. Children were more likely to attempt to calculate Doubles facts and Tens facts, and more children used recall strategy for these facts.

For K2 children, Adding 0 was the easiest group of facts, as the average time spent on these facts was 3.27 seconds, and 91% of K2 children could successfully arrive at the correct answers. Eighty-seven percent of the K2 children could correctly calculate the Adding 1, 2, 3 facts, while 81% of them could reach the right answers for Doubles. The most difficult ones were Bridging ten facts, as there were only 64% of K2 children who could correctly compute these facts. Children seemed to be confident in doing Doubles with addends no bigger than 5. There were more than 50% of K2 children who could give the answers within 2 seconds (86% for $1+1$, $2+2$, and $5+5$; 68% for $3+3$; 50% for $4+4$). Comparatively, $1+1$ and $5+5$ are the easiest facts to K2 children, whereas with K1 children, they were $1+1$ and $2+2$.

All children in K3 could reach correct answers when solved Adding 0 facts and Tens facts. Bridging 10 and Near double facts were facts with which K3 children made the most errors. Children used the derived facts strategy most frequently to solve Bridging 10 and Near Double facts.

Overall, Bridging 10 was found to be the most difficult group of facts for all three year levels of children in this study (K1, K2, and K3).

Discussion and Conclusion

The results of this study appear to suggest that formal instruction has a strong impact on children's strategy choice for basic addition fact computation. Six strategies identified in the literature, namely, Sum, Min, Counting on, Recall, Derived facts and guessing were all evident as used by the kindergarten children in this study. However, one further strategy not discussed in the literature, but used by children in this study, was the abacus as the children had been taught how to use an abacus since their first year in kindergarten.

The findings also indicate that there is a developmental sequence in addition strategy use. Children who made attempts to calculate the facts in this study predominantly used guessing and the sum strategy in their first year of kindergarten. In K2, children began to use diverse strategies to compute different facts; however, they are efficient in sum strategy and use recall strategy very often. Use of the abacus strategy was also employed more by K2 children than K1, but at the same time, they still relied on Sum. When children entered K3 level, they could recall most of the facts. However, they depended on Visualised abacus strategy most often to calculate the facts that they were not confident to recall. It was also noteworthy that the K3 children did not use their fingers or other concrete materials to compute addition facts as they also felt ashamed to do so.

It is interesting that only few children frequently used the min strategy in this kindergarten. It seems reasonable to suggest that the Min strategy period may not be an important transition period for those children to develop more efficient addition strategies. The visualised abacus strategy was a very dominant strategy in this study. These data strongly suggest that instruction dominated by use of the abacus has supported the development of number sense and some understanding of number relationships.

Reference

- Baroody, A. J. (1987). The development of counting strategies for single-digit addition. *Journal for Research in Mathematics Education*, 18(2), 141-157.
- Braten, I. (1998). *Cognitive strategies in mathematics, Part I: On children's strategies for solving simple addition problems*.
- Carpenter, T. P., & Moser, J. M. (1982). The development of addition and subtraction problem-solving skills. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education*, 15(3), 179-202.
- Carr, M., & Jessup, D. (1994, April). *Gender differences in first grade mathematics strategy use: Social, metacognitive, and attributional influences*. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans.
- Christensen, C. A. (1991). Instruction, practice, and children's use of thinking strategies to solve basic addition fact. In G. Evans (Ed.), *Learning and teaching cognitive skills*. Melbourne: Australian Council for Educational Research.
- Christensen, C. A., & Cooper, T. J. (1991). Children's solution strategies to single-digit addition problems: Some Australian data. *Educational Research and Perspectives*, 18(2), 62-70.
- Griffin, S. (2003). Laying the foundation for computational fluency in early childhood. *Teaching Children Mathematics*, 9(6), 306-309.
- Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review*, 79, 329-343.
- Heffelfinger, T., & Flom, G. (2004, 2007). *Abacus: Mystery of the bead*. Retrieved December 13, 2007, from <http://webhome.idirect.com/~totton/abacus/pages.htm#Addition1>
- Houlihan, D. M., & Ginsburg, H. P. (1981). The addition methods of first and second-grade children. *Journal for Research in Mathematics Education*, 12, 95-106.
- Isaacs, A. C., & Carroll, W. M. (1999). Strategies for basic-facts instruction. *Teaching Children Mathematics*, 5(9), 508.
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental Mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.
- McIntosh, A., & Dole, S. (2000). Mental computation, number sense and general mathematics ability - are they linked? In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, pp. 401-408). Sydney: MERGA.
- Parkman, J. M., & Groen, G. J. (1971). Temporal aspects of simple addition and comparison. *Journal of Experimental Psychology*, 89, 335-342.

- Postlewait, K. B., Adams, M. R., & Shih, J. C. (2003). Promoting meaningful mastery of addition and subtraction. *Teaching Children Mathematics*, 9(6), 354-357.
- Resnick, L. B. (1989). Developing mathematical knowledge. *American Psychologist*, 44, 162-169.
- Siegler, R. S. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology: General*, 116(3), 250-264.
- Siegler, R. S. (2003). Implications of cognitive science research for mathematics education. In J. Kilpatrick, Martin, W.B., & Schifter, D.E. (Ed.), *A research companion to principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Siegler, R. S., & Campbell, J. (1989). Individual differences in children's strategy choices. In P. L. Ackerman, R. Sternberg, J & R. Glaser (Eds.), *Learning and individual differences: Advances in theory and research*. New York: W. H. Freeman and Company.
- Siegler, R. S., & Jenkins, E. (1989). *How children discover new strategies*: Hillsdale, NJ. Lawrence Erlbaum Associates.
- Suppes, P., & Groen, G. J. (1967). Some counting models for first grade performance data on simple addition facts. In J. M. Scandura (Ed.), *Research in mathematics education*. Washington, DC: National Council of Teachers of Mathematics.
- Zhang, D. (1998). Some characteristics of mathematics education in East Asia. Retrieved from [http:// www.math.admu.edu.ph/tsg22/Zhang.html](http://www.math.admu.edu.ph/tsg22/Zhang.html)